through, starting from an insertion loss function of allowed form. His procedure parallels that used in our numerical example. He does not introduce a complex variable equivalent to p, however, and thus does not have Richards' theorem available for proving physical realizability. In particular, he makes no point of the second condition for the physical realizability of an impedance function. Ozaki and Ishii,[11] clearly state this second condition, but they do not parallel Darlington by starting from a given insertion loss function. E.M.T. Jones in the 1956 IRE CONVENTION RECORD uses a complex variable, but he makes no mention of the second condition for physical realizability, and appears, in his proof of physical realizability, to have appealed to Richards for a theorem which Richards did not prove.

[11] H. Ozaki and J. Ishii, "Synthesis of transmission-line networks and the design of uhf filters," IRE TRANS., vol. CT-2, p. 325–336; December, 1955.

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An Analysis of the Diode Mixer Consisting of Nonlinear Capacitance and Conductance and Ohmic Spreading Resistance*

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Summary—A method is presented for calculating the mixer admittance matrix Y' which results when an ohmic impedance is connected in series with a diode mixer described by an admittance matrix Y. There are no restrictions on the frequency dependence of the ohmic impedance nor on the number of harmonic sidebands considered. The equations are worked out in detail for the "low Q" case in which signal, image, and intermediate frequencies are considered, and it is shown that Y' in this case is "nearly low Q." As a result of this analysis the usual criterion for good high-frequency mixing, *i.e.*, that the product of the spreading resistance and the barrier capacitance be small compared with unity, is criticized and a new figure of merit is proposed.

Explicit formulas have been derived for calculating the elements of Y' when Y represents the parallel combination of a nonlinear conductance and capacitance. In general, these formulas are cumbersome, but three special cases have been considered in detail.

Case 1: Zero spreading resistance and equal admittances connected to image and signal terminals. Results: a) The conversion gain is independent of the contact area. b) Regions of negative IF conductance are always associated with arbitrarily high gain.

Case 2: High-frequency, small spreading resistance, image shorted across nonlinear conductance and capacitance. Results: a) The conversion loss and the IF admittance can be given by closed equations. b) The IF conductance can be negative. c) Regions of negative IF conductance are bounded by regions of arbitrarily small IF conductance. d) The conversion loss can *decrease* with increasing frequency. e) Low conversion loss is accompanied by narrow bandwidth.

Case 3: The spreading resistance is zero and the image is shorted. Results: a) Above a certain frequency negative IF conductance is obtained and arbitrarily low conversion loss is possible. b) The situation is quite similar to that of Case 1.

Measurements of mixer performance at the "available terminals" are discussed and the failure of the "phenomenological theory of mixing" as a basis for making such measurements is emphasized.

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INTRODUCTION

THIS PAPER will be concerned principally with the mixing properties of the circuit of Fig. 1 (next page), where arrows indicate that g and C are functions of the voltage across them. Frequent reference will be made to Torrey and Whitmer[1] and whenever possible the notation used therein will be followed here.

The circuit of Fig. 1 has been widely used, qualitatively at least, as an equivalent circuit for point-contact crystal diodes,[2] particularly for microwave work in which the capacitor is of importance. The part of the crystal diode that Fig. 1 is supposed to represent is shown in Fig. 2. The terminals are at the dotted lines AA' and BB'. The distance from the line AA' to the surface is a small fraction of the shortest wavelength involved, while the line BB' is located so as to include nearly all of the spreading resistance. It can be shown that the latter requirement will be fulfilled if BB' is several times the contact diameter away from the contact region.

The validity of the circuit of Fig. 1 as a representation of Fig. 2 is open to question. It has been verified in the

[1] H. C. Torrey and C. A. Whitmer, "Crystal Rectifiers," McGraw-Hill Book Co., Inc., New York, N. Y.; 1948.
[2] *Ibid.*, p. 24.

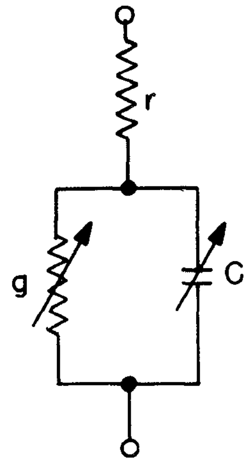

Fig. 1—Small-signal equivalent circuit.

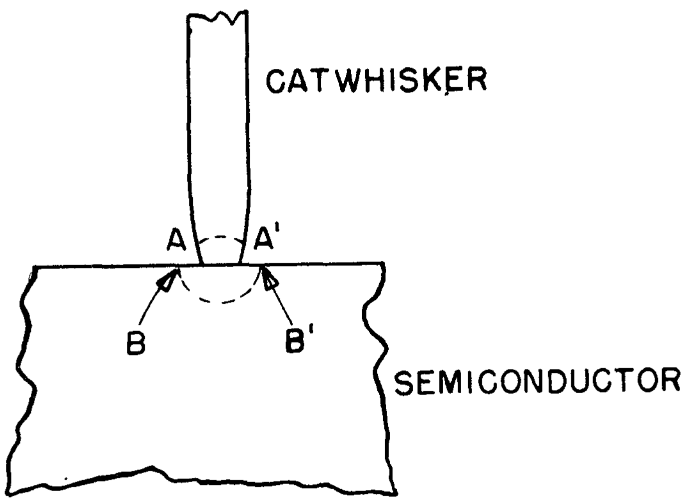

Fig. 2—Point-contact crystal diode.

several megacycle range for silicon point-contact crystals. Analysis of the mixer properties of this circuit yields negative IF conductance under certain conditions. This has been observed in H. Q. North's welded-contact crystals.[3] The actual equivalent circuit is probably at least as complicated as Fig. 1.

The region under consideration is clearly only part of the crystal diode; in addition, there is the rest of the whisker and semiconductor and various components necessary for proper support and protection. These components, however, are nearly lossless and are certainly linear so that the effect is simply that of an impedance transformer and need not be explicitly considered. In choosing the terminals discussed above we have included the region of the nonlinearity and the region of the important losses. The use of the "low-frequency" concepts of voltage, current, and lumped components is justified, if, to put it roughly, the distance between AA' and BB' is small compared with a wavelength. For commercial crystals designed for a minimum wavelength of about 4 cm, the contact diameter is less than 0.001 inch, so that the condition is well met.

[3] *Ibid.*, p. 398.

The central engineering problem of a microwave mixer is to relate the small-signal parameters to quantities of interest when the crystal is operated as a mixer —decidedly a large-signal nonlinear problem. The behavior (except for noise considerations) of the mixer is completely specified when values are obtained for the elements of the mixer admittance matrix. The situation is much like that encountered in the theory of linear n poles, except that each terminal pair is associated with a different low-power-level frequency. Since an infinite number of these frequencies exists in the mixer, the admittance matrix is an infinite one. Once the admittance matrix is known, the conversion loss, IF admittance, etc., can be calculated when specified admittances terminate each pair except the IF terminal pair.

The complete circuit of Fig. 1 has not (to the author's knowledge) been quantitatively treated in the literature. The case in which C is assumed linear has been treated rather thoroughly.[4] The case in which r is zero but both g and C are nonlinear has been partially treated,[5] but only with respect to the spectacular effect of negative IF conductance which the nonlinear C makes possible; there is in this reference no calculation of conversion loss and no quantitative information on the effect of spreading resistance. Only the broadband (signal and image admittances equal) case was treated by Torrey and Whitmer. One of the purposes of this paper is to point out that at sufficiently high frequencies, the nonlinear capacitance in Fig. 1 can have an important effect on the properties of the mixer even though negative IF conductance does not appear. Negative IF conductance may not appear for one or both of the following reasons: C is not sufficiently nonlinear, or the spreading resistance is too large. The properties of the barrier conductance also have some influence.

Treatment of Spreading Resistance

In a mixer there are an infinite number of frequencies at low power levels. It is customary to consider only three of these· the signal, intermediate, and image frequencies.[6] This approximation becomes increasingly poor for low-loss mixers. Since the inclusion of nonlinear capacitance can result in a mixer with zero conversion loss, the general treatment will be for any number of low-level signals.

A generalization of Fig. 1 may easily be made (in principle) as follows: the spreading resistance r may be considered complex and may be different for each low-level frequency of interest. he t"spreading impedance" will therefore be denoted by z_n, where n denotes the frequency.

[4] *Ibid.*, p. 157.
[5] *Ibid.*, p. 406.
[6] *Ibid.*, p. 111.

This might be useful if, for example, one wished to include the catwhisker (which has a different impedance at signal than at IF frequencies because of its inductance and skin effect) in the spreading impedance.

Assume that the admittance matrix Y of the mixer not including the z_n's, is known. Now, Y may be anything, and will at first be kept general, but the case of most interest at present is that in which Y is the admittance matrix of the circuit of Fig. 1 with $r=0$. The latter is given by Torrey and Whitmer[7] for the most general case (arbitrary number of terminal pairs, arbitrary LO voltage waveform) and specialized to a consideration of signal, image, and intermediate frequencies only,[8] and with the local oscillator voltage (LO) waveform restricted to even functions.

The frequencies of interest can be expressed as $\beta+n\omega$ where β is the intermediate frequency, n is a positive or negative integer, and ω is the LO frequency. Current and voltage at the frequency $\beta+n\omega$ will be denoted by i_n and v_n, respectively. Then $I=YV$, where I and V are column matrices formed from the i_n's and the v_n's.

If i_n' and v_n' are the new values after the addition of z_n, then $i_n'=i_n$ and $v_n'-i_n'z_n=v_n$. These equations are substituted into $I=YV$ and the i_n''s are solved for in terms of the v_n''s. The coefficients of the v_n''s are the elements of the new admittance matrix Y' which describes the original matrix with the spreading impedance added.

The calculation is straightforward and the results can be given as follows: form the diagonal matrix Z whose elements along the diagonal are the z_n's. Next form the matrix $B=ZY+1$, where 1 denotes the unit matrix. Calculate b, the determinant of B, and all of its cofactors b_{rc}, where r and c denote row and column. Then, y_{rc}', a typical element of Y', will be given by

$$y_{rc}' = \left(\sum_i b_{ir} y_{ic} \right) \Big/ b,$$

where the unprimed y's are the elements of Y.

To illustrate the procedure, and for future reference, the equations will be worked out for the case of the signal, image, and intermediate frequencies in the notation of Torrey and Whitmer.

At this point Y is arbitrary and can be written

$$Y = \begin{bmatrix} y_{\alpha\alpha} & x_{\alpha\beta} & y_{\alpha\gamma} \\ y_{\beta\alpha} & y_{\beta\beta} & y_{\beta\gamma} \\ y_{\gamma\alpha} & x_{\gamma\beta} & y_{\gamma\gamma} \end{bmatrix}. \tag{1}$$

Assume, as is customary, that the z_n's are all real and equal. Then

$$Z = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} \tag{2}$$

$$B = ZY + 1 = \begin{bmatrix} 1 + y_{\alpha\alpha}r & y_{\alpha\beta}r & y_{\alpha\gamma}r \\ y_{\beta\alpha}r & 1 + y_{\beta\beta}r & y_{\beta\gamma}r \\ y_{\gamma\alpha}r & v_{\gamma\beta}r & 1 + y_{\gamma\gamma}r \end{bmatrix} \tag{3}$$

and

$$Y' = \begin{bmatrix} y_{\alpha\alpha}' & y_{\alpha\beta}' & y_{\alpha\gamma}' \\ y_{\beta\alpha}' & y_{\beta\beta}' & y_{\beta\gamma}' \\ y_{\gamma\alpha}' & y_{\gamma\beta}' & y_{\gamma\gamma}' \end{bmatrix} \tag{4}$$

where

$$\begin{aligned}
by_{\alpha\alpha}' &= y_{\alpha\alpha}b_{11} + y_{\beta\alpha}b_{21} + y_{\gamma\alpha}b_{31} \\
by_{\alpha\beta}' &= y_{\alpha\beta}b_{11} + y_{\beta\beta}b_{21} + y_{\gamma\beta}b_{31} \\
by_{\alpha\gamma}' &= y_{\alpha\gamma}b_{11} + y_{\beta\gamma}b_{21} + y_{\gamma\gamma}b_{31} \\
by_{\beta\alpha}' &= y_{\alpha\alpha}b_{12} + y_{\beta\alpha}b_{22} + y_{\gamma\alpha}b_{32} \\
by_{\beta\beta}' &= y_{\alpha\beta}b_{12} + y_{\beta\beta}b_{22} + y_{\gamma\beta}b_{32} \\
by_{\beta\gamma}' &= y_{\alpha\gamma}b_{12} + y_{\beta\gamma}b_{22} + y_{\gamma\gamma}b_{32} \\
by_{\gamma\alpha}' &= y_{\alpha\alpha}b_{13} + y_{\beta\alpha}b_{23} + y_{\gamma\alpha}b_{33} \\
by_{\gamma\beta}' &= y_{\alpha\beta}b_{13} + y_{\beta\beta}b_{23} + y_{\gamma\beta}b_{33} \\
by_{\gamma\gamma}' &= y_{\alpha\gamma}b_{13} + y_{\beta\gamma}b_{23} + y_{\gamma\gamma}b_{33}.
\end{aligned} \tag{5}$$

It is usually assumed that $y_{\beta\beta}$ is real and that there are no circuits *in* the mixer which can discriminate between the signal frequency and the image frequency, *i.e.*, $y_{\gamma\beta}=y_{\alpha\beta}^*$, $y_{\beta\gamma}=y_{\beta\alpha}^*$, $y_{\gamma\alpha}=y_{\alpha\gamma}^*$, and $y_{\gamma\gamma}=y_{\alpha\alpha}^*$, where the asterisks denote complex conjugates. If the preceding four conditions are all met, one says, by definition, that the mixer, or the mixer admittance matrix, is "low Q."[9] Then,

$$Y = \begin{bmatrix} y_{\alpha\alpha} & y_{\alpha\beta} & y_{\alpha\gamma} \\ y_{\beta\alpha} & g_{\beta\beta} & y_{\beta\alpha}^* \\ y_{\alpha\gamma}^* & y_{\alpha\beta}^* & y_{\alpha\alpha}^* \end{bmatrix}.$$

It can then be shown that $b_{33}=b_{11}^*$, $b_{31}=b_{13}^*$, $b_{32}=b_{12}^*$, $b_{21}=b_{23}^*$, and b_{22} is real. From these relations and (5), it follows that $y_{\alpha\beta}'=y_{\gamma\beta}'^*$, $y_{\alpha\alpha}'=y_{\gamma\gamma}'^*$, $y_{\gamma\alpha}'^*=y_{\alpha\gamma}'$, $y_{\beta\alpha}'=y_{\beta\gamma}'^*$, and $by_{\beta\beta}'$ is real. Since b is not, in general, real, $y_{\beta\beta}'$ is not real and the mixer is not low Q. A mixer which meets all the requirements of a low Q mixer except that $y_{\beta\beta}'$ be real will be termed "nearly low Q." If the time zero is properly chosen,[10] $y_{\beta\alpha}'$ can be made real and then

$$Y' = \begin{bmatrix} y_{\alpha\alpha}' & y_{\alpha\beta} & y_{\alpha\gamma}' \\ g_{\beta\alpha}' & y_{\beta\beta}' & g_{\beta\alpha}' \\ y_{\alpha\gamma}'^* & y_{\alpha\beta}'^* & y_{\alpha\alpha}'^* \end{bmatrix}.$$

There would seem to be little point in presenting the long expressions for the y''s but, since each b_{ij} appears several times in Y', the following formulas may be

[7] *Ibid.*, p. 165.
[8] *Ibid.*, p. 408.
[9] *Ibid.*, p. 115.
[10] *Ibid.*, p. 117.

useful

$$\begin{aligned} b_{11} &= (1 + g_{\beta\beta}r)(1 + y_{\alpha\alpha}^*r) - r^2 y_{\alpha\beta}^* y_{\beta\alpha}^* \\ b_{13} &= r^2 y_{\beta\alpha} y_{\alpha\beta}^* - y_{\alpha\gamma}^* r(1 + g_{\beta\beta}r) \\ b_{12} &= r^2 y_{\beta\alpha}^* y_{\alpha\gamma}^* - r y_{\beta\alpha}(1 + y_{\alpha\alpha}^* r) \\ b_{21} &= r^2 y_{\alpha\gamma} y_{\alpha\beta}^* - r y_{\alpha\beta}(1 + y_{\alpha\alpha}^* r) \\ b_{22} &= |1 + y_{\alpha\alpha}r|^2 - r^2 |y_{\alpha\gamma}|^2 \\ b &= (1 + y_{\alpha\alpha}r)b_{11} + y_{\alpha\beta}r b_{12} + y_{\alpha\gamma} r b_{13}. \end{aligned} \tag{6}$$

Finally, in order to compute Y' for the circuit of Fig. 1, one uses for Y the formulas in Torrey and Whitmer[11] for the general case (*LO* waveform arbitrary), or those for the case of *LO* waveform "even."[12]

At microwave frequencies the terminals to which Y' refers are not available for measurement. If for example, one is dealing with a waveguide mixer, impedance measurements must be made in the waveguide at some distance (so that higher modes will die out) from the terminals to which Y' refers. In addition, there will be "parasitic" impedances associated with the mechanical mounting of the point on the crystal. This problem may be present to a lesser extent for the intermediate frequency. One can formally handle this by connecting a four-pole (usually nearly lossless) on each of the terminal pairs. A new admittance matrix will result, Y'', which describes the mixer at terminals which are actually available for measurement. It is intuitively clear that if all the four-poles are linear and if the four-poles connected to the signal and image terminals cannot discriminate between these two frequencies, Y'' will be nearly low Q. The proof would be quite similar to the above proof that Y' is nearly low Q and has been given in essence by Peterson and Llewellyn.[13] The fact that these authors were treating the case of the linear capacitance is immaterial.

Qualitative Statements Concerning the Effect of Finite Spreading Resistance

There is a frequently employed argument that for good high-frequency performance, ωCr should be much less than one. In the first place, since the barrier capacitance is a function of bias, it is not clear what is meant by C in the above expression. Presumably one is to assume that the barrier capacitance is linear. Even so, the above criterion does not appear to be useful or accurate. Consider the two extreme instances, the first in which ωCr is small because r is very small while ωC is finite, and the second in which ωCr is small because ωC is very small, while r is finite. If the above criterion is correct, these two cases should give about the same high-frequency performance. In the first case, the low-frequency conversion loss will be low (assume that in both cases the barrier conductance is sufficiently nonlinear), and the high-frequency conversion loss will be identically low. This is true because of the general theorem that a reactive network placed in tandem with a four-pole does not change its conversion loss. Or, if proof for a particular case is desired, (15) in this paper and (17) in Torrey and Whitmer,[14] may be used to calculate the conversion loss L. It will be noted that ωC_0, where C_0 is the linear capacitance, appears only in the combination $\omega C_0 + b_a$ and since b_a is arbitrary, L will be independent of frequency. In the second instance, however, L will be very high for both the low-frequency and the high-frequency case.

The case of the linear capacitor is treated in Torrey and Whitmer,[15] and the equations for the elements of Y' are also given.[16] The quantity ωCr does appear repeatedly, but this is not the whole story by any means since r appears in other ways also. This must be so, because obviously, in the low-frequency case, in which ωC_0 is negligible, large r will impair the performance.

One would like to enunciate a rule or rules of thumb for low L for the circuit of Fig. 1 at all frequencies. It is clear that the requirement on r is that the elements of Y' be nearly equal to the elements of Y. Eq. (5) indicates that the requirements on b and the b_{ij}'s are that b and b_{11} be close to one and that b_{13}, b_{21}, and b_{12} be nearly zero. Eq. (6) shows that the only conditions which will simultaneously satisfy the above conditions are that $y_{ij}r$ be small compared with one. Using (6), one obtains the result that $g_i r \ll 1$ and $\omega C_i r \ll 1$. Since g_1 and g_2 must be equal to or less than g_0, and C_1 and C_2 must be equal to or less than C_0, it is sufficient to require that $g_0 r$ and $\omega C_0 r$ be small compared with one. Adding conditions for nonlinearity, one can say that at low frequencies the requirements are $g_1 g_2 / g_0^2 \cong 1$ and $g_0 r \ll 1$ while for high frequencies the *additional* requirements are $C_1 C_2 / C_0^2 \cong 1$ and $\omega C_0 r < 1$. Perhaps it is not too far fetched to propose, for an over-all wideband figure of merit, that $g_1 g_2 C_1 C_2 / g_0^3 C_0^3 r^2$ be large compared to one.

For the point-contact geometry, r increases as the diameter of contact decreases while the C_i's and g_i's decrease with the area of contact.[17] This is a possible explanation for the well-known fact that, particularly for high frequencies, small contacts are necessary for low L. It is worth pointing out that this is due completely to the geometry of the point contact which is roughly planar for the C_i's and g_i's but three-dimensional for r. In a completely planar geometry, for example, L would be independent of the cross-sectional area.

[11] *Ibid.*, p. 166.
[12] *Ibid.*, p. 408.
[13] L. C. Peterson, and F. B. Llewellyn, "The performance and measurement of mixers in terms of linear-network theory," Proc. IRE, vol. 33, pp. 458–476; July, 1945.

[14] Torrey and Whitmer, *op. cit.*, p. 409.
[15] *Ibid.*, p. 157.
[16] *Ibid.*, p. 160.
[17] *Ibid.*, p. 98.

OPERATIONAL MEANING OF THE ELEMENTS OF Y''

Probably the most straightforward and elegant way to measure and express the properties of a mixer is by means of the elements of Y''. This information, after all, completely describes the properties of the mixer at a given bias and *LO* drive, just as the elements of the admittance matrix of any linear n pole described its properties. Equipped with this information the engineer can calculate the conversion loss, the IF impedance, and other properties as a function of the signal and image termination.

In Torrey and Whitmer[18] there is a discussion of what is called the Phenomenological Theory of Conversion (PTC). According to this theory, one can calculate the elements of Y'' for the case of signal, image, and intermediate frequencies from a series of measurements of the rectifying properties of the crystal made at *LO* power levels; the theory does not appear to be applicable to the mixer which includes a nonlinear capacitance. During the war almost all the measurements of the elements of Y'' were based on the PTC, and at the present time the JAN specifications are based on this method. Nevertheless, the method is highly questionable, particularly if the capacitance is nonlinear. It would therefore seem pertinent to outline a general method for measuring the elements of Y'', regardless of the equivalent circuit of the mixer. The equations for mixer IF admittance y_β and for mixer conversion loss L are[19]

$$y_\beta = Y_{\beta\beta} - \frac{Y_{\alpha\beta}Y_{\beta\alpha}}{Y_{\alpha\alpha} + y_a} \tag{7}$$

and

$$L = Wg_\beta \tag{8}$$

where

$$W = \frac{|y_a + Y_{\alpha\alpha}|^2}{g_a |Y_{\beta\alpha}|^2}. \tag{9}$$

As before, α, β, and γ refer to the signal, intermediate, and image frequencies, respectively; the Y_{ij}'s are the elements of the 2×2 admittance matrix Y_2, which is formed from the 3×3 matrix Y'' by specifying a termination at the image terminals; y_a is the signal admittance; g_a is the real part of y_a; and g_β is the real part of y_β. The relationships between the elements of Y'' and Y_2 are[20]

$$Y_{\alpha\alpha} = y_{\alpha\alpha} - \frac{y_{\alpha\gamma}y_{\alpha\gamma}^*}{y_{\alpha\alpha}^* + y_c^*} \tag{10}$$

$$Y_{\alpha\beta} = y_{\alpha\beta} - \frac{y_{\alpha\gamma}y_{\alpha\beta}^*}{y_{\alpha\alpha}^* + y_c^*} \tag{11}$$

$$Y_{\beta\alpha} = y_{\beta\alpha} = \frac{y_{\alpha\gamma}^* y_{\beta\alpha}^*}{y_{\alpha\alpha}^* + y_c^*} \tag{12}$$

$$Y_{\beta\beta} = y_{\beta\beta} - \frac{y_{\beta\alpha}^* y_{\alpha\beta}^*}{y_{\alpha\alpha}^* + y_c^*} \tag{13}$$

where y_c is the admittance at the image terminals. Note that the capital letters indicate the elements of Y_2 and the small letters indicate the elements of Y''. For convenience, the elements of Y'' are not primed.

It is known[21] that $y_{\beta\alpha}$ can be made real by a proper choice of the origin of time. If $\boldsymbol{y}_{\beta\alpha}$ is a particular value of $y_{\beta\alpha}$ for some particular choice of time zero, then the general form of $y_{\beta\alpha}$ is: $y_{\beta\alpha} = \exp(-jwt_0)\boldsymbol{y}_{\beta\alpha}$ where t_0 is arbitrary. In the same way, $y_{\alpha\gamma} = \boldsymbol{y}_{\alpha\gamma}\exp(2jwt_0)$ and $y_{\alpha\alpha} = \boldsymbol{y}_{\alpha\alpha}$. By substituting into (12), the following general expression for $Y_{\beta\alpha}$ is obtained: $Y_{\beta\alpha} = \exp(-jwt_0)\boldsymbol{Y}_{\beta\alpha}$, where $\boldsymbol{Y}_{\beta\alpha}$ is a value of $Y_{\beta\alpha}$ for some choice of time zero. Now $\boldsymbol{Y}_{\beta\alpha}$ is in general complex, but by proper choice of t_0, $Y_{\beta\alpha}$ can be made real. In what follows it is assumed that $Y_{\beta\alpha}$ is real and it will be denoted by $G_{\beta\alpha}$.

The parameters to be measured are the elements of Y''. The general procedure is as follows: first, set y_c at some known value, y_c^I. Second, measure y_β for three different known values of y_a. Eq. (7) can now be used to solve for $Y_{\beta\beta}^I$, $Y_{\alpha\alpha}^I$, and $Y_{\alpha\beta}^I G_{\beta\alpha}^I$, the I's corresponding to the value of y_c. Third, L and g_β are measured for any known value of y_a, and from (8) and (9), $G_{\beta\alpha}^I$ is obtained. L can be measured by any reliable method such as the heterodyne method[22] using either a cw or noise signal, and g_β can be measured with an rf bridge. Thus, a complete set of Y_{ij}'s is obtained. The above procedure is repeated for a different value of y_c and a second set of Y_{ij}'s is obtained. This is more than enough information to solve for the elements of Y'' by using (10) through (13) twice.

The practical measurement is complicated by the fact that y_a and y_c are not easily varied independently. This is because the waves associated with the two admittances are located physically in the same transmission line. Thus one must resort to frequency-sensitive terminations in order to vary y_a and y_c independently. It appears that an excellent pair of choices for y_c would be infinity and zero, for then $Y_{ij}^I = y_{ij}$ and the only remaining unknown would be $y_{\alpha\gamma}$. Then from (11)

$$Y_{\alpha\beta}^{II} = y_{\alpha\beta} - \frac{y_{\alpha\gamma}y_{\alpha\beta}^*}{y_{\alpha\alpha}^*}$$

$y_{\alpha\gamma}$ can be obtained. The above method would appear to be practical. The values infinity and zero for y_c can be approximated by inserting a transmission type cavity in the line tuned to the signal frequency. Variation of y_a could be accomplished either by varying a termina-

[18] *Ibid.*, p. 119.
[19] *Ibid.*, p. 136.
[20] *Ibid.*
[21] *Ibid.*, p. 116.
[22] *Ibid.*, p. 200.

tion on the far side of the cavity, or by detuning the cavity (y_a would have to be measured with, for example, a standing-wave machine). In the case of y_c, it could probably be assumed that the corresponding standing-wave ratio would be infinite. Note that no microwave impedance measurements at the microwatt level are necessary. Since the author has never made measurements of this type, he cannot vouch for the success of the above method. Nevertheless, it clearly demonstrates the operational meaning of the elements of Y''.

Calculations of L, y_β, and g_β

Once the admittance matrix Y' has been calculated by the procedure given under the section on "Treatment of Spreading Resistance," (7), (8), and (9) can be used to calculate L, y_β, and g_β. It is clear that for the general case, these equations will be extremely complicated. There is an additional complication owing to the fact that, in general, one wishes these quantities as a function of y_a and y_c. It would appear quite feasible, however, to make numerical calculations, particularly if a computer is available, and it is planned to do so. However, if one chooses highly idealized cases some manageable equations can be obtained.

Case 1

Assume that $r=0$, and that the image and signal are terminated with equal admittances (broadband case). Insofar as g_β is concerned, this case is treated in Torrey and Whitmer.[23] For convenience, Y, which in this case is equal to Y', is reproduced here:

$$Y' = \begin{bmatrix} g_0 + j\omega C_0 & g_1 + j\omega C_1 & g_2 + j\omega C_2 \\ g_1 & g_0 & g_1 \\ g_2 - j\omega C_2 & g_1 - j\omega C_1 & g_0 - j\omega C_0 \end{bmatrix}. \quad (14)$$

There are some additional relationships which are not mentioned in Torrey and Whitmer: $C_1 \leqq C_0$, $C_2 \leqq C_0$, and $2C_1^2/C_0 \leqq C_0 + C_2$. These relations hold for the same reasons that the equivalent ones hold for g_0, g_1, and g_2.[24]

The statement is frequently made that when the frequency is high, the area of the point contact must be made small in order to reduce the capacitance. This is not true, however, when $r=0$. The C_i's and the g_i's are so intimately related to each other that a decrease in contact area, which would reduce the C_i's would reduce the g_i's by the same factor. In other words, g_i/C_i depends only on the character of the potential barrier and not on the geometry. Now L is a dimensionless quantity which depends on the g_i's, the C_i's, and y_a which is arbitrary. Assume that for some geometry, y_a is adjusted for minimum conversion loss, L_1. Now assume that the geometry is changed (*e.g.*, the contact area is reduced) so as to halve the C_i's and the g_i's. Then assuming that one readjusts y_a to minimize the conversion loss again to the value L_2, L_1 and L_2 will be equal.

Torrey and Whitmer point out that for negative g_β the barrier capacitance must be strongly nonlinear, *i.e.*, C_1/C_0 or C_2/C_0 must be large. It should be noted, however, that L depends on all the elements of the admittance matrix and that therefore as long as C_1 and C_2 are finite there will be some high frequency at which the nonlinearity of the barrier capacitance will play a role in determining L. For example, from $y_{\alpha\beta}$, (14), no matter how small C_1 is there will be some frequency at which ωC_1 will be of the same order of magnitude as g_1.

It is clear that Y', (14), leads to absurd results if g_1 or g_0 are set equal to zero. This is a result of the assumption in Torrey and Whitmer that the intermediate frequency β is such that $\beta C_i \ll g_i$. This restriction, which is usual in microwave mixers, may be removed if desired. This has been done in some recent work at the Bell Telephone Laboratories.[25] Only the case of the pure nonlinear capacitance is considered, but some new effects show up which are not in evidence when βC_i is assumed negligibly small compared with g_i. Throughout this paper it is assumed that $\beta C_i \ll g_i$.

As Torrey and Whitmer point out, the region on the y_a plane corresponding to negative g_β adjoins regions of either arbitrarily large g_β and/or arbitrarily small g_β. In the latter case, since W remains finite, it is clear that L is arbitrarily small in the same region. One would like to know the behavior of L in the region in which g_β is arbitrarily large. The condition for $g_\beta = \infty$ is $B_0^2 + G_0^2 = g_2^2 + \omega^2 C_2^2$, where $G_0 = g_0 + g_a$, $B_0 = b_0 + \omega C_0$.[26] Since Torrey and Whitmer do not give an equation for W, it is presented here.

$$W = \frac{1}{g_a} \frac{[1 - (g_2^2 + \omega^2 C_2^2)/(G_0^2 + B_0^2)]^2 (G_0^2 + B_0^2)^2}{g_1^2[(G_0 - g_2)^2 + (B_0 - \omega C_2)^2]}. \quad (15)$$

It is seen that W goes to zero for the same condition. If, however, one calculates the product $g_\beta W$ the following expression is obtained:

$$L = W g_\beta = MN \frac{G_0^2 + B_0^2 - (g_2^2 + \omega^2 C_2^2)}{(G_0^2 + B_0^2)^2} \quad (16)$$

where

$$M = g_0 \left\{ \left[b_a + \omega \left(C_0 - \frac{g_1 C_1}{g_0} \right) \right]^2 + \left[g_a + g_0 \left(1 - \frac{g_1^2}{g_0^2} \right) \right]^2 - G^2 \right\} \quad (17)$$

$$N = \frac{(G_0^2 + B_0^2)^2}{g_a g_1^2 [(G_0 - g_2)^2 + (B_0 - \omega C_2)^2]} \quad (18)$$

[23] *Ibid.*, p. 408.
[24] *Ibid.*, p. 409.

[25] M. C. Waltz, A. E. Bakanowski, and A. Uhlir, "Second Interim Technical Report on Crystal Rectifiers," Bell Telephone Labs., Task 8, Signal Corps Contract DA 36-039 sc-5589; January 15, 1955.
[26] Torrey and Whitmer, *op. cit.*, p. 409. See (15).

and G is defined in Torrey and Whitmer.[27] Thus, the condition in question gives an infinite g_β but a conversion loss of zero. One can now make the statement that for the broad-band case, negative g_β regions are always associated with arbitrarily high gain, regardless of whether g_β goes to infinity or to zero at the boundary of the negative g_β region. Note the use of the term "arbitrarily small." The mere fact of conversion gain, *i.e.*, $L<1$, is not particularly significant since mixers are not connected in tandem to obtain a large over-all gain. The quantity of interest is the over-all receiver noise figure which is given by $F_r=L(t+F_i-1)$, where F_r is the over-all receiver noise figure, t is the noise temperature ratio of the mixer, and F_i is the noise figure of the IF amplifier. It is clear that a mixer with $L=1.05$ is very little worse than one with $L=0.95$ in spite of the fact that the latter mixer shows conversion gain. A mixer which can have an arbitrarily small L is quite a different matter however. The important question is then what happens to t when L is small. At present this question cannot be answered without resorting to experiment.

Case 2

Assume that r is finite, but that the image can be shorted at the barrier. This is impossible in principle if r is frequency independent, since $1/r$ would be the largest admittance which could be connected to the image terminals. The requirements for approximating the above are given by (92)–(95) in Torrey and Whitmer,[28] and it can be shown that these requirements are reasonable if $1/r \gg \omega C_0$. Obviously at sufficiently high frequencies this condition will not be met. Assume that the frequency is such that $g_i \ll \omega C_i$. The above two assumptions are not mutually exclusive. Assume the maximum nonlinearity, $g_1=g_0=g$, $C_1=C_0=C$. Finally, assume that $gr \ll 1$. The last condition is almost always met. Then[29,30]

$$Y=\begin{bmatrix} Y_{\alpha\alpha} & Y_{\alpha\beta} \\ Y_{\beta\alpha} & Y_{\beta\beta}\end{bmatrix}=\begin{bmatrix} j\omega C & j\omega C \\ g & g\end{bmatrix} \tag{20}$$

$$B=\begin{bmatrix} 1+j\omega Cr & j\omega Cr \\ rg & 1\end{bmatrix}. \tag{21}$$

$$\left.\begin{aligned} b &= 1-j\omega Cr; \\ b_{11} &= 1; \\ b_{12} &= -rg; \\ b_{21} &= -j\omega Cr; \\ b_{22} &= 1+j\omega Cr. \end{aligned}\right\} \tag{22}$$

$$Y'=(1/b)Y. \tag{23}$$

$$L=g_\beta W \tag{24}$$

where

$$W=\frac{1}{g_a}\frac{|y_a+y_{\alpha\alpha}'|^2}{|y_{\beta\alpha}'|^2} \tag{25}$$

$$W=\frac{1}{g_a}\frac{|by_a+j\omega C|^2}{g^2} \tag{26}$$

$$W=\frac{1}{g_a}\,\frac{g_a^2[1+(\omega Cr)^2]+b_a^2[1+(\omega Cr)^2]-g_a(2\omega^2C^2r)+b_a(2\omega C)+(\omega C)^2}{g^2}. \tag{27}$$

$$y_\beta=\frac{g}{b}-\frac{j\omega Cg/b^2}{y_a+\dfrac{j\omega C}{b}} \tag{28}$$

$$y_\beta=\frac{1}{b}\left(g-\frac{j\omega Cg}{by_a+j\omega C}\right). \tag{29}$$

For zero spreading resistance

$$W^0=\frac{1}{g_ag^2}\left[g_a^2+(b_a+\omega C)^2\right] \tag{30}$$

$$y_\beta^0=g\left(1-\frac{j\omega C}{y_a+j\omega C}\right) \tag{31}$$

$$g_\beta^0=\mathrm{Re}\,(y_\beta^0)=g\left[1-\frac{(\omega C)^2+\omega Cb_a}{g_a^2+b_a^2+2b_a\omega C+(\omega C)^2}\right]. \tag{32}$$

If we are interested in small values of L, we can focus our attention on g_β since W cannot approach zero or infinity for any nontrivial case. Consider the condition for $g_\beta=0$:

$$g_a^2+(b_a-0.5\omega C)^2=0.25(\omega C)^2. \tag{33}$$

On the y_a plane, this is the equation of a circle with center at 0, $-0.5\ \omega C$ and radius $0.5\ \omega C$. If W does not approach zero, and it does not, then points outside the circle and arbitrarily close to the circle will give arbitrarily small conversion loss. Points inside the circle yield negative IF conductance which presumably is to be associated with IF oscillations, and the equation for L does not apply. The situation is quite similar to the broad-band case.[30] It is presented here because to the author's knowledge it has not appeared in the literature and yet the equations are shorter and it is a simpler case to understand than the broad-band one. A consideration of L as a function of frequency will yield some interesting results. Suppose that one considers C and g to be fixed and ω and y_a to be such that the point y_a is a considerable distance away from the circle. Then L will be given by Wg_β and will be large if y_a is far enough away from the circle. However, as the frequency is increased while y_a is held constant, the conversion loss will de-

[27] *Ibid.*, p. 409.
[28] *Ibid.*, p. 136.
[29] *Ibid.*, (92)–(95); and also, p. 408 (11).
[30] *Ibid.*, p. 409. Note that in the narrow-band high-frequency case presented above, the region of negative g_β is adjacent only to a region of small g_β and not to a region of large g_β. Note also that W possesses no nontrivial singularities.

crease, approaching zero as the circle approaches y_a. Thus, we have a conversion loss that decreases as the frequency increases. It is quite clear that one pays for small L in terms of bandwidth. For y_a must be reasonably close to the circle for small L, but a change in frequency will in general move y_a with respect to the circle, not only because the circle moves but also because in general y_a is also a function of frequency. These two effects could conceivably cancel over a small-frequency band. If y_a moves inside the circle then the oscillatory condition holds and if y_a moves too far away from the circle, L increases.

Returning to the case $r \neq 0$, let us set $y_a' = y_a b$ and define a function

$$f(y) = g - \frac{j\omega C g}{y + j\omega C}. \tag{34}$$

Then from (31),

$$y_\beta^0(y_a) = f(y_a), \qquad r = 0 \tag{35}$$

while from (29),

$$y_\beta = \frac{1}{b} f(y_a'), \qquad r \neq 0. \tag{36}$$

Note that y_β^0 is considered a function, while y_β is not. Then

$$y_\beta = \frac{1 + j\omega Cr}{1 + (\omega Cr)^2} g \left[1 - \frac{j\omega C}{g_a' + j(b_a' + \omega C)} \right]. \tag{37}$$

Of course, y_a is arbitrary; *i.e.*, we assume y_a is under the control of the circuit engineer and can be made to assume any value. The question arises as to whether or not y_a' is also arbitrary, for if it is, the effect of the spreading resistance on y_β is simply multiplication by the factor $1/b$. Now

$$y_a' = g_a' + jb_a' = (1 - j\omega rC)(g_a + jb_a);$$

$$g_a = \frac{g_a' - \omega C r b_a'}{1 + (\omega Cr)^2}; \tag{38}$$

$$b_a = b_a' + \omega Cr \left[\frac{g_a' - \omega C r b_a'}{1 + (\omega Cr)^2} \right]. \tag{39}$$

From (38), some values of b_a' will require a negative g_a for solution. This is not physically acceptable; however, if b_a' is limited to negative values, a positive g_a will always satisfy (38). Assuming that b_a' is limited to negative values, (29) gives

$$y_\beta = \frac{1}{1 + (\omega Cr)^2} (1 + j\omega Cr) [g_\beta^0(g_a', b_a') + jb_\beta^0(g_a', b_a')] \tag{40}$$

and

$$g_\beta = \frac{1}{1 + (\omega Cr)^2} [g_\beta^0(g_a', b_a') - \omega C r b_\beta^0(g_a', b_a')] \tag{41}$$

where

$$y_\beta^0(y_a') = g_\beta^0(g_a', b_a') + jb_\beta^0(g_a', b_a') = f(y_a'). \tag{42}$$

The factor $1/[1 + (\omega Cr)^2]$ affects the magnitude of g_β but is not a factor in determining whether or not g_β becomes negative. From (31)

$$g_\beta = \frac{1}{1 + (\omega Cr)^2} \left[g_\beta^0(g_a', b_a') + \frac{(\omega C)^2 g g_a' r}{g_a'^2 + (b_a' + \omega C)^2} \right]. \tag{43}$$

This equation shows, and it is quite clear intuitively, that the presence of finite spreading resistance always decreases the possibility of obtaining negative g_β, and shows precisely the condition for negative g_β with finite spreading resistance in terms of the value of g_β without spreading resistance. Of course, g_β is given by (32). Eqs. (8), (27), and (43) can be used to calculate L when g_β is positive.

Case 3

Suppose $r = 0$ and the image is shorted. Then[31]

$$Y = Y' = \begin{bmatrix} g_0 + j\omega C_0 & g_1 + j\omega C_1 \\ g_1 & g_0 \end{bmatrix} \tag{44}$$

and, from (7)

$$\frac{g_\beta}{g_0} = 1 - \frac{(g_1^2/g_0)(g_0 + g_a) + (\omega C_1 g_1/g_0)(\omega C_0 + b_a)}{(g_0 + g_a)^2 + (\omega C_0 + b_a)^2}. \tag{45}$$

To investigate the conditions for $g_\beta = 0$, the numerator and denominator of the fraction are set equal, and with a little manipulation the following equation is obtained:

$$\left[g_a + \left(g_0 - \frac{g_1^2}{2g_0} \right) \right]^2 + \left[b_a + \left(\omega C_0 - \frac{\omega C_1 g_1}{2g_0} \right) \right]^2 = \frac{g_1^2}{4g_0^2} (g_1^2 + \omega^2 C_1^2). \tag{46}$$

This is the equation of a circle in the y_a plane with center at

$$\left(\frac{g_1^2}{2g_0} - g_0 \right), \quad \left(\frac{\omega C_1 g_1}{2g_0} - \omega C_0 \right)$$

and radius squared of

$$\frac{g_1^2}{4g_0^2} (g_1^2 + \omega^2 C_1^2).$$

Note that both coordinates of the center of the circle are always negative. The general condition for g_β to be negative with a physically realizable y_a is that the radius be greater in magnitude than the x coordinate of the center of the circle. Thus a low-frequency cutoff for negative g_β can be calculated by setting the radius of the circle equal to the x coordinate of the center of the circle. Some special cases are of interest. For the case of a linear capacitor, $C_1 = 0$, the center of the circle is at $(g_1^2/2g_0) - g_0$; $-\omega C_0$ and the radius squared is

[31] *Ibid.*, p. 408.

$$\frac{g_1^4}{4g_0^2}.$$

Since $g_1 \leqq g_0$, all points inside the circle can be shown to correspond to negative g_a, and this is physically inacceptable. Just as in the broad-band case, then[32] negative IF conductance is not obtained with a linear capacitor. Note, however, that one of the conditions, which is certainly not met in practice, is that the *LO* voltage waveform be symmetrical. To the author's knowledge, the more general case has not been investigated, although there is experimental evidence[33] that relaxation of this restriction will not change the picture.

[32] *Ibid*

[33] H. Q. North, *et al.*, "Welded Germanium Crystals," GE Rep., Contract OEMsr-262, Order No. DIC 178554; September 20, 1945.

If one sets $g_0 = g_1 = g$ and $C_0 = C_1 = C$, *i.e.*, maximum nonlinearity for both the barrier capacitance and conductance, the center of the circle is at $-0.5\,g$, $-0.5\,\omega C$ and the radius is $0.5\sqrt{g^2+\omega^2C^2}$. Thus, in this special case there is no low-frequency cutoff for negative IF conductance.

When g_β is not negative, W is of interest and is given by

$$W = \frac{(g_0 + g_a)^2 + (b_a + \omega C_0)^2}{g_a g_1^2}. \qquad (47)$$

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Resonance Properties of Ring Circuits*

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Summary—**The ring guide or ring circuit, a microwave device consisting of a waveguide having the ends connected to form an annular ring, has properties similar to those of ordinary resonant cavities. Wave propagation within the ring guide, its interaction with a waveguide to which it is coupled, and its resonant circuit properties are investigated in this report. The properties of a prototype circuit consisting of a ring guide of rectangular cross section were found to agree with theory.**

INTRODUCTION

CAVITIES with conducting walls have, at wavelengths of the order of their geometrical sizes, the same electromagnetic properties as resonant circuits consisting of capacitances and inductances at lower frequencies. Therefore, cavities are useful as resonant circuits and filter elements at microwave frequencies. They can also be considered as waveguide sections short circuited at each end. The electromagnetic energy oscillates between the electric and magnetic states. Standing waves and an imaginary Poynting vector are of significance at any point in the cavity.

A new type of microwave circuit[1] consists of a waveguide having the ends connected to form a ring in which waves progressing in one direction only are excited. This circuit is characterized by a real Poynting vector within the ring guide cavity. Properties of the circuit, including the form of wave propagation within the ring, interaction with a waveguide to which it is coupled, and its Q-value, are investigated. Circuit performance when excited to produce traveling waves in one direction is compared with that obtained when excited to produce waves in both directions.

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[1] F. J. Tischer, Swedish Patent No. 152,491; August 26, 1952.

WAVE PROPAGATION IN THE RING CIRCUIT WHEN COUPLED TO A WAVEGUIDE

The system under investigation consists of a waveguide to which a ring guide is coupled as shown schematically in Fig. 1. In order to obtain waves progressing in one direction only in the ring, directional coupling is used. When nondirectional coupling is applied, waves progressing in both directions are obtained. These two types of coupling are shown in Fig. 2, *i.e.*, directional coupling by two holes spaced a distance of $\lambda_g/4$ apart and nondirectional coupling by a single hole.

In the following derivation based on the wave concept, h indicates the waves progressing toward the coupling element, while r corresponds to waves reflected and traveling from the coupling element. The symmetry plane AA', Fig. 2, is used as phase reference. The ports of the main waveguide are 1 and 2, while those of the secondary guide which will be connected to form a ring are 3 and 4. The other parameters which describe the wave propagation in the region of the junction are the reflection coefficients ρ_{nn} at the four ports and the transmission coefficients T_{nm} between the different ports with reference to plane AA'.